Godzilla Geometry a.k.a. the Cosmic Distance Ladder

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Abstract: The distance ladder is a fundamental tool of astronomy and yet still very abstract. The activity described uses a digital camera to empirically determine the relationship of relative size to distance and then extends this concept to astrophysical objects accessible via the SDSS Skyserver.

See: http://cfcp.uchicago.edu/education/explorers/2003summer-YERKES/#day

Introduction: The combination of simple consumer technology (i.e., a digital camera and a computer) and hands on laboratory activities can offer students a visceral feel for how the cosmological distance ladder works. This physical and direct experience helps to convince students that it is possible to measure large objects and distances indirectly. Once this conceptual framework has been established the distance ladder can be developed with real astronomical data. The context of where this experiment was first tried and the student laboratory experience are described below. Photos of the experiment and the original, downloadable, student lab hand-out can be found at: http://cfcp.uchicago.edu/education/innercity/yerkes-summer2003.html#day

This laboratory activity was first developed for a Yerkes Summer Institute (YSI) in 2003. YSI is a residential science immersion program that is one facet of the larger Space Explorers Program, which is a Signature Program of the Kavli Institute for Cosmological Physics. The Space Explorers Program is a multi-year multi dimensional commitment to inner-city minority middle and high school students. In addition to YSI the Space Explorers participate in weekly laboratory sessions on campus, enrichment trips, faculty lectures and laboratory tours, and a Winter Institute at Yerkes Observatory. By immersing these students in the process of doing science, we hope to increase their interests and abilities in science and math and help them to succeed in high school and beyond. Graduates of the Space Explorers matriculate in college as science majors at a rate that is 500% better than is predicted by combined national and Chicago Public Schools statistics. See: http://cfcp.uchicago.edu/education/explorers/index.html. Godzilla Geometry was one of three daytime laboratories developed for YSI 2003. At YSI the students were divided into three small groups and spent the better part of one day working on each experiment. Evenings were spent with laboratory activities that took advantage of the observatory's resources. After all the students had cycled through the daytime experiments, another
small group, comprised of students from each lab group, extended the exercise and reported on the laboratory to their peers, their parents and younger students.

**The Laboratory Experience:**
Part I – Empirically Determining the Relationship Between Apparent Size & Distance
Part II - Testing This Empirically Determined Relationship
Part III – Reinforcing Concepts & Using the Relationship for Predictions
Part IV – Extending the Concept to Astronomical Objects & Building a Distance Ladder

**Part I – Empirically Determining Relationship Apparent Size & Distance:**
Students took digital photos of known objects at known distances (e.g. 20-500 feet) and then determined their apparent size in pixels. At Yerkes the known objects were an inflatable Godzilla and a standard golf flag. The Godzilla was used because it is fun and because of the role of apparent size in monster movies. The golf flag was chosen because there is a golf course adjacent to the observatory, which allowed for additional experiments at larger distances. Student challenges included indexing the images, and accurately measuring large distances. (Note: It is important that the camera settings remain fixed, with auto focus off).

Data analysis consisted of determining the number of vertical pixels (e.g., apparent size) of the known objects at the observed distances. During YSI the students also gathered images with the observatory telescopes, and so we chose to use Hands-On Universe (HOU) software for consistency. HOU was developed to handle the standard astronomical FITS file format. This meant that it was necessary to convert the jpeg’s output by the digital camera to FITS files which is somewhat odd, but lead to greater continuity. In general I would recommend working with the more common jpeg’s and any commercial software that can be used to count pixels. Student challenges involving the data analysis included developing a criterion for edges, using the software, and graphing the results. It is also possible to do this analysis as a facilitated group activity with a computer attached to a projector.
Sample Student Results: Note how at the large distances/small apparent sizes the relationship is roughly linear ~1/r (e.g., twice as far and half as many pixels/half the apparent size). However, this relationship (i.e., the small angle approximation) breaks down dramatically at smaller distances (e.g., ~100 feet or an apparent size of 100 pixels).

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Part II – Testing the Empirically Determined Relationship:

This portion of the laboratory involved gathering data on “unknowns” that could latter be measured and compared to the predictions of the empirical graph. The students photographed the golfing Godzilla and marked the location. The distance to Godzilla was then determined by comparing the apparent size (e.g., the number of pixels) to the graph and seeing what distance this corresponded to. The actual distance was then measured to demonstrate the validity of the method. The second “unknown” was the height of the ninety-foot diameter dome that houses the great 40-inch refractor. A photo was taken of the dome with Godzilla at the base. By comparing their relative apparent sizes the students were able to estimate the height of the dome. This exercise demonstrated some of the limitations of our experimental method and of our simple model. (e.g., Some of the more advanced students noted that the top of the dome was about 45 feet further away then Godzilla.)

Part III - Reinforcing the Concepts:

These exercises moved towards greater abstractions while cementing the basic concepts. One challenge involved predicting the relative positions of a smaller model Godzilla and a human to make them appear the same size, as one would need to do for a monster movie. The second challenge more closely modeled the astronomical distance ladder rungs. A golf flag on the adjacent golf course, that was at a much greater
distance then the students had yet to measure, was photographed under the same conditions as used in the earlier part of the laboratory. The students were able to estimate the distance to the far away flag by extrapolating their empirical size-distance graph, given the assumption that all golf flags are the same size (analogous to assuming that globular clusters or galaxies are the same size). Once the golfers were gone the students measured the true distance.

Part IV - Extending the Concepts to Astrophysical Objects & the Cosmic Distance Ladder:

After the students had demonstrated that it was possible to indirectly measure size and distance, we shifted to astronomical data from the Sloan Digital Sky Survey. First the distance to M31 was determined by assuming that globular clusters in the Milky Way Galaxy and in M31 are the same true size e.g., that their separation can be estimated by their apparent size differences. Next M31 was used as a stepping-stone to even greater distances. The Space Explorers compared the apparent sizes of M31 and galaxies in a distant cluster of galaxies. By assuming that the true size of a galaxy in the cluster is roughly the size of M31 they were able to estimate relatively how much further away the cluster was. Finally the students were given the true distance to one of the globular clusters in the Galaxy that they started with, and were able to use that information along with the relative distances to determine absolute distances.

Conclusion: The cosmological distance ladder is a complex abstraction both in terms of the enormous scales and distances it encompasses and the predictive powers of a mathematical model. The Godzilla Geometry experiment provided middle and high school students with a direct physical experience constructing and testing a predictive model (e.g., an empirical graph of distance Vs. apparent size). The Space Explorers’ tangible experience with the powers and limitations of a local distance ladder made the cosmological distance ladder more approachable and comprehensible. By directly testing a distance ladder that worked on scales of hundreds of feet, the students were better able to understand the power of mathematical models and the scales of the cosmos. The physical laboratory experience transformed the distance ladder from an abstraction that must be believed to something that they had directly tested and had a gut feeling for.

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Student Laboratory Handout:
Godzilla Geometry a.k.a. Distance Ladder
(Instructors: Randy Landsberg, Mark SubbaRao, Dan Holz, & Takemi Okamoto)

The further away something is the smaller it appears. Think of the skyscrapers in the Loop that make up the Chicago skyline. From five or so miles away in Hyde Park, enormous structures like the Hancock Building, one of the tallest buildings in the world, appear smaller than modest local landmarks. From the observation deck on the 93rd floor of the Hancock Building the people at street level look like ants. Our brains help us to make sense of these illusions, movie makers use them for special effects, and scientists take advantage of them to determine distances to far away objects.

GODZILLA (1954 - 1975) / aka: Gojira

| Height: 50 meters (164 feet) |
| Mass: 20,000 metric tons (22,000 tons) |
| Fight record: 18 wins, 3 losses, 7 ties |
| Powers/weapons: Atomic ray, super regenerative power (Godzilla can be wounded, but his G-cells heal very rapidly) |
| Year of first appearance: 1954, in Godzilla |
| Origin: Monster brought to life by Atomic testing in the Pacific Ocean during the '50's. source: http://www.avia-art.com/godzillawebsite/ginfo.html |

It is possible to use this effect to measure distance because the apparent change in size of an object is related to how far away that object is and can be well described with mathematics. There are some rules to follow (e.g., you need to look at the object the same way each time). In this laboratory we will empirically determine how apparent size changes with distance for everyday objects (and movie monsters) on the grounds of Yerkes. We will then use our measurements, in a way very similar to how astronomers do, to indirectly determine distances (and relative sizes).

This laboratory and Night Lab-3, the Distance Ladder, are meant to be complementary. They both demonstrate that one can determine distances and relative sizes, without having to actually travel to the object being measured, and build on earlier measurements to develop a distance ladder that takes one further and further beyond one’s reach.

Crossing the Digital Divide: Connecting Pixels to Angles
In this laboratory we will use a digital camera, specifically a Nikon Coolpix 4500. When a digital camera takes a picture, the image is divided up into a grid of tiny identically sized and shaped pixels or picture elements. Digital cameras naturally divide images into lots of tiny pixels because light coming through the camera’s lens falls on a charge-coupled device (CCD), which is physically divided into many pixels. The CCD chip in the Nikon 4500 has 4 mega pixels, or 4 million pixels. The dimensions of the CCD are 2,272 pixels wide by 1,704 pixels tall (note 2,272 x 1,704 = 3,871,488 is rounded up to 4 million by optimistic advertising).
Astronomers like to measure objects in terms of angular scale (most often very very very small angular scales); the objects they observe are very very very far away. Angular size is a very useful measurement because one does not need to know exact distances or sizes to make useful measurements, and because it is easy to compare images from one telescope to another.

Figure: Side view of same height object observed from near and far. Note: when the object is near the angular size ($\phi_1$) is a much bigger angle than the apparent angular size when it is far ($\phi_2$)

A bigger angular size translates to a bigger image hitting more pixels. So the object of height $h$ will take up more pixels in the near image (corresponding to angular size $\phi_1$) than in the far away picture (with smaller angular size, $\phi_2$). If the camera set-up is kept constant, then the number of pixels and the angular size are directly proportional, to a very good approximation:

$$\phi \times \text{constant} = \text{number of pixels}.$$

It will be simpler for us to determine the relationship between distance and the apparent size of an object measured in number of pixels, rather than in degrees.

Fortunately, in many astronomical situations the object is so distant and so tiny that we can use a shortcut called the small angle approximation. When the conditions are right to use the small angle approximation size is directly proportional to distance. That is, if something looks half as big, it is twice as far (assuming that the something is either same object or two objects that are the same size). We cannot assume that the small angle approximation will work for the distances and objects that we will be working with. Instead we will measure the relationship of apparent angular size to distance.

For the extra curious:

The vertical angular field of view of the Nikon Coolpix 4500 can be determined by taking photos of an object of a known height from a known distance. We now know two sides of a right triangle. Trigonometry tells us that if we know the length of the opposite and adjacent sides of a right triangle then we can find the size of the angle

$$\tan(\phi) = \frac{\text{opposite}}{\text{adjacent}}$$

for example for the figure above $$\tan(\phi_1) = \frac{h}{d_1}, \text{ so}$$

$$(\phi)(\text{radians}) = \arctan \left( \frac{\text{opposite}}{\text{adjacent}} \right)$$

$$(\phi_1)(\text{radians}) = \arctan \left( \frac{h}{d_1} \right)$$

Now we know the angular scale of the object. By counting the number of pixels that it takes up in the image we now know the size in terms of angular scale and number of pixels.
We can equate them: \( \text{angular size } \phi = \text{(number of pixels)} \) and divide to find the conversion factor: \#pixels/degree.
Under the conditions that this lab will use (e.g., focus set at infinity, zoom all the way in) we have determined that the vertical angular field of view of the Nikon 4500 is about (45 degrees). This means that it takes about 38 of the 1,704 vertical pixels to correspond to an angular size of one degree. (e.g., expect about 19 vertical pixels for a full moon).

**Determine How Apparent Size Changes With Distance:**
In this part of the lab you will take digital pictures of a golf flag and a model Godzilla, objects of known heights, from different distances and measure how their apparent sizes change.

Hardware: Digital Camera, (LCD projector), long tape measure, standard objects (golf flag & model Godzilla of known heights), dark ground cover for contrast, computers with pcmica slot, HOU software and JPG to FITS file converter.

**IMPORTANT:** it is critical that the camera is set up the same way each time otherwise the auto zoom will change the perspective. The camera should be set to infinity (mountain peak icon) and the zoom all the way in (i.e. no zoom). The camera should also be mounted in a fixed fashion to the tripod.

Step 1 – Measure the heights of Godzilla & the Golf Flag. Record in your lab notebook.

Step 2 - Take photos of Godzilla and the golf flag from different distances (from ~10 to 300 meters)
Your group will need to develop a system to:
   a) To determine the distance and
   b) To index the photos (e.g., which photo is from which distance)

Step 3 – Analyze your data
Your lab instructors will demonstrate how to download the images on to the computers and then how to use the software:
   a) To convert the files from JPG to FITS format and
   b) To determine how many pixels tall each object is
      • Measure the height of Godzilla and the golf flag for each distance and record these values in your lab notebook.
      • Create a data table for both objects (distance, number of pixels).
      • Record in your notebook any relevant details of how you determined where Godzilla or the golf flag began and ended.

Step 4 – Graph your data
   • Generate a graph of apparent size (pixels) vs. distance (meters).
   • Comment in your lab notebook on anything you observe in this graph.

Next you will use the results of your Godzilla/Golf Flag apparent size vs. distance measurements to determine the distances to far away objects and their sizes. As a group you will pick challenges and agree on experiments to solve them. Record your method, observations and results in your lab notebook.
Distance & Size Challenge – Distance to the Dome? Height of the Dome?
Use a model Godzilla of known height, and the distance/apparent size relationship developed earlier to determine how far away from the big dome you are at any point on the lawn and to determine how tall the dome is. Then measure this distance directly and compare your results. Compare your height estimate to the know height.

Distance Challenge – How far to that flag?
Determine the distance to a far away golf flag on the George Williams golf course using the distance vs. angular size relationship developed earlier. Then measure this distance directly (when no one is golfing) and compare your results. Assume all golf flags are the same height.

Monster Movie Challenge – How to make the Model Godzilla appear as big as the dome in the camera?
Use the apparent size/distance relationship and the height of the dome to estimate how to position the camera, the model and the dome to create a perspective that makes Godzilla appear about the same height as the dome. Test your prediction with the camera. Are you ready for Hollywood?

Image Challenge
Climb up the cosmological distance ladder by comparing the sizes of similar extragalactic objects in images from the Sloan Digital Sky Survey.

First, based on the assumption that globular clusters belonging to the Milky Way and M31 are of comparable size (or luminosity), determine the relative distance to M31.

Next, assuming that the galaxies in a distant cluster are like M31 in size, find the relative distance to that cluster.

Finally, translate these observations into absolute distances - you will be given the distance to the original globular cluster.

Some Useful Conversion Factors

1 meter = 3.2808 feet

360° in a circle

1° = 60 arcminutes = 60’
1 arcminute = 1’ = 60 arcseconds = 60”

so
there are \(60\text{(arcmin/deg)} \times 60\text{(arcsec/arcmin)} = 3,600\text{ arcseconds per degree}\)

1 arcsecond = 1” = 1/3600°